Developing Critical Reflection and Inquiry Among Teacher Candidates in an Elementary Mathematics Methods Course

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Developing Critical Reflection and Inquiry Among Teacher Candidates in an Elementary Mathematics Methods Course

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Abstract

This study explores how past and present experiences with mathematics interact as teacher candidates engage in a series of critical inquiry and reflection opportunities embedded in a mathematics methods course. The study involved 28 teacher candidates enrolled in an undergraduate education program within a large university. Data were collected through a series of written reflections and semi-structured interviews. The results reveal that the teacher candidates possess a wide variety of feelings towards mathematics, yet they are often influenced by how they were taught as students. Further, negative experiences with mathematics appear to be more prevalent than positive ones. Opportunities for critical inquiry and reflective thinking allowed the participants to develop new understandings about teaching and learning mathematics. Overall, the findings indicate that there are meaningful pedagogical opportunities for giving teacher candidates opportunities to revisit previous experience, construct new meaning, and challenge old assumptions about teaching and learning mathematics.
Introduction

Exploration, play, and joy are important aspects of mathematics teaching and learning. And yet, they are—quite often—not what students and teachers typically associate with math class in PK-12 classrooms. Type the word “math” into Google Images, and the results are almost comical: image after image of chalkboards filled with endless mathematical scribbles. A few scrolls down, one comes upon a unique drawing of a quizzical Homer Simpson, finger poised against chin, in front of the same chalkboard of numerical scribbles—some joy starting to enter the pictures in this mathematical realm. Further on, there are cutesy images: personified rulers smiling broadly, colorful “Math is fun!” posters perfectly sized for classroom bulletin boards, and school-aged children staring up at a pleasantly drawn forest of numbers. But one would likely be on an endless search to find a visual representation of what math in elementary, middle, and high school really could—or should—be: a collaborative inquiry among a few young minds with question marks, frustration, and fear erased by curiosity, deep questioning, and wondering. As researchers and teacher educators, our goals with regard to mathematics instruction for teacher candidates revolve around the ideals of inquiry and exploration. This article presents findings from a study that examined the experiences and perspectives of teacher candidates enrolled in a mathematics methods course that explicitly foregrounded inquiry and exploration.

Review of Associated Literature

We are in an era of transformation in the preparation and assessment of teacher candidates. As teachers and their school leaders are increasingly being held accountable for providing quality teaching and learning experiences, various states have begun to establish new standards and utilize new performance-based assessments in order to improve the quality of the teachers entering the profession (Roberts-Hull et al., 2015). Despite these changes, the current state of teacher preparation in the United States has been characterized as well below what is desired and needed (National Comprehensive Center for Teacher Quality, 2010). Research indicates that many first-year teachers do not feel confident or well prepared to teach in the classroom (Levine, 2006). For example, data from the Schools and Staffing Survey of the National Center for Education Statistics indicates that only one-fifth of new teachers felt prepared enough to teach curriculum materials and assess their students’ learning (National Comprehensive Center for Teacher Quality, 2010). In addition, PK-12 students demonstrate a lack of proficiency in core subject areas—such as mathematics—as documented in international and national assessments (NAEP, 2018; OECD, 2015), which has echoed concerns raised by new teachers and brought up questions about the effectiveness of teacher preparation (Boyd et al., 2009; Levine, 2006; Wiseman, 2012).

Recent studies identify teachers as key agents of educational change and student outcomes (Beckman et al., 2004; Clarke & Clarke, 2004; Fullan, 1993; Hattie, 2003; Krainer, 2011; Tattoo et al., 2008). For example, in an effort to assess the quality of mathematics teaching in U.S. classrooms and document how variations in the quality of teaching might produce different student outcomes, Rivkin et al. (2005) found that students whose teachers provided high-quality instruction gained
1.5 grade equivalents, while students whose teachers provided low-quality instruction only made a gain of 0.5 grade equivalents during the same academic year. While this research reported on the effects of quality of instruction on student achievement in a single school year, Sanders and Rivers (1996) studied the cumulative effects of the quality of mathematics teaching and reported that both ineffective and effective teachers have additive and cumulative effects on student achievements in mathematics. Further, these effects are often not compensatory. That is, the disparities in student performance as a result of teaching qualities show a persistent pattern or have an enduring effect either for better or for worse (Sanders & Rivers, 1996).

Although the education system depends on teachers to help young children learn math concepts and develop foundational math understanding, the same system does not put enough effort into equipping teachers with the necessary mathematics knowledge and skills that they require to undertake the task. Research indicates that most of the teacher education programs in higher education do not offer more than one course specifically devoted to mathematics teaching and learning, which is not enough to equip prospective teachers with the domain-specific knowledge that they need in order to provide quality mathematics education (Copple, 2004; Ginsburg et al., 2004; Ginsburg et al., 2008).

Therefore, it is not surprising that numerous studies have indicated that many teacher candidates do not possess the mathematical knowledge that is necessary to provide quality mathematics teaching and learning opportunities to young children (Ball, 1990; Ma, 1999; Hill et al., 2004) and often do not feel confident in teaching mathematics (Bursal & Pazkanos, 2006; Copley, 2004; Wilkins, 2008). Burton et al. (2008) tie this lack of competence and confidence in elementary mathematics teaching and learning to teacher preparation when they state:

\textit{Most often, mathematics content courses address mathematics concepts and procedures, while mathematics methods courses focus on learning theories and the application of activities to facilitate learning. There is little opportunity for future elementary educators to explore the specific mathematical content they will be teaching and even less time to explore it in the manner in which they are being expected to teach. (p.1)}

Further discussing how teachers’ content knowledge can affect the quality of their instructional practices, Brophy (1991) states:

\textit{Where [teachers’] knowledge is more explicit, better connected and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond fully to student comments and questions. Where their knowledge is limited, they will tend to depend on the text content, de-emphasize interactive discourse in favor of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352)}

Researchers have also examined how elementary students’ low levels of mathematics achievement are highly correlated with not only their teachers’ mathematics knowledge (Ball, 1990; Beckman et al., 2004; Clarke & Clarke, 2004; Seidel & Shavelson, 2007) but also their approach toward the subject (Ball, 1990). Since both content knowledge and beliefs play a fundamental role in teachers’
effectiveness (Ball, 1990; Baumert et al., 2010; Shulman, 1986), the preparation of teacher candidates presents a crucial opportunity to develop future teachers’ confidence and understanding of mathematical content knowledge. Further, research also suggests that teachers’ pedagogical approaches are an important aspect of teacher competence, influencing teachers’ instructional behaviors and their students’ motivation (Klassen et al., 2011; Klassen & Tze, 2014; Tschannen-Moran et al., 1998; Tschannen-Moran & Woolfolk Hoy, 2001). In order to build teacher candidates’ confidence and competence in teaching mathematics, teacher preparation programs need to explore new ways to provide learning and teaching experiences in mathematics that will positively impact teacher candidates’ knowledge, beliefs, and feelings about mathematics teaching and learning (Ball, 1990; Battista, 1986; Baumert et al., 2010).

Changing the Landscape

Teacher education is crucial to the development of teacher candidates’ pedagogical content knowledge and beliefs about the way mathematics should be taught. As Sheridan (2016) states, “Teacher education can either reinforce or challenge pedagogical beliefs, providing the opportunity to preservice teachers to create new versions of firmly held truths” (p.3). Therefore, it is critical for teacher educators to facilitate a reflective practice that helps teacher candidates to actively consider and reconsider beliefs and practices that improve their pedagogical ability (Moon, 2004). This will also allow teacher educators to understand how teacher candidates create meaning from their experiences (Darling-Hammond, 2010; Richardson, 1990).

Researchers acknowledge that reflective teaching promotes teachers’ professional development and improves the quality of teaching and learning (Endacott & Sturtz, 2015; Paakkari et al., 2015; Rossum & Hamer, 2010; Vosniadou et al., 2008; Wubbels, 1992). Researchers in this field point to the difficulty of changing robust beliefs (Korthagen et al., 2006; Paakkari et al., 2015), while others conclude that such a change is possible when certain experiences are afforded in the preservice education setting. In order to construct new beliefs, teacher candidates must be afforded opportunities to link theory to practice within existing preconceptions (Wubbels, 1992), engage in critical reflection (Endacott & Sturtz, 2015), and participate in analyzing and reflecting on pedagogical reasoning (Vosniadou et al., 2008).

Reflection is a mode of thought historically associated with creating “warranted assertabilities” (Dewey, 1938, p.15) from experience. In the context of mathematics teaching and learning, we believe that the practice of reflective teaching results in: (a) gaining personal and professional knowledge in mathematics; (b) creating room for teacher candidates to conceive of new perspectives; and (c) inviting teacher candidates to challenge old assumptions with regard to mathematics. Teacher education coursework can afford an opportunity for teacher candidates to reflect upon their beliefs, challenge attitudes, and grow in content knowledge once they are in the classroom (Ball, 1990; Battista, 1986).
Unfortunately, critical reflection does not come naturally to most teacher candidates; therefore, appropriate opportunities for reflection should be provided as part of their teacher education preparation (Yang, 2009). The literature presented in this review suggests that teacher candidates come to the classroom with negative beliefs around mathematics and take those beliefs with them to their classrooms, in particular feeling underprepared and/or incompetent to provide quality mathematics instruction. In this study, we used our role as teacher educators to infuse our mathematics methods course with several reflective thinking and learning opportunities, aiming to explore how our students interact with and respond to these opportunities as they developed their abilities regarding teaching and learning mathematics. In developing this inquiry, we were interested in exploring how these experiences might allow our students to be more open to receiving the experiences provided throughout the class and what the students thought of these experiences. In this way, our research contributes to the ongoing discourse on the value and impact of reflective practice in mathematics education for teacher candidates. It also explores how teacher candidates’ reflections that were focused on their previous experiences with mathematics enabled the teacher candidates to construct new meaning and challenge old assumptions.

Research Context and Design

The participants for this study were seniors in an undergraduate education program in a large university in a metropolitan city in the northeastern United States. The participants consisted of 28 students who were enrolled in an elementary-level mathematics methods course that foregrounded inquiry and reflection. This course is the second course in a two-part sequence. In the first course of the sequence, students explore mathematics methods for elementary-grade classrooms. In this second course, the students explore the mathematical content more deeply, building stronger foundations for how to teach mathematical concepts in ways that promote what Jo Boaler (2015) has termed a mathematical mindset. In order to build and develop a mathematical mindset, we deliberately structured the course so that students investigated and reflected on the ways that mathematical content strands (e.g., number and operations, measurements, algebraic thinking, geometry, and data analysis) can be examined in a manner that is constructivist and engaging.

Participant Information

In order to recruit participants for this study, we offered our students an overview of the study details at the first class session. We explained that data would be collected from existing course structures and assignments, and we further explained that there would be no penalty for any student who chose to opt out. After reviewing the Participant’s Statement of Rights, all of the students across the two sections of our mathematics methods classes agreed to be in the study and provided a signed consent form. Of the total participants for this study, 27 were female and 1 was male. The majority of the participants were within the age range of 20–23, with only two participants over the age of 25. Of the total participants, 67% identified as White, 14% identified as Asian, 14% identified as Latina, and 1% identified as African American.
Data Sources and Collection

This study was designed to explore participants’ experiences within a mathematics methods course that was purposely designed to grow and foster critical reflection and inquiry with regard to teaching and learning mathematics. To that end, we sought answers to the following research questions:

1) In what ways does a mathematics methods course that foregrounds inquiry and reflection provide space for teacher candidates to develop their own critical reflection and inquiry around teaching and learning mathematics?
2) What are the participants’ perceptions and experiences about teaching and learning mathematics after participating in a mathematics methods course that foregrounds inquiry and reflection?
Data Collection—Phase One

Data were collected in two phases over the course of one academic year. In the first phase, we collected a series of written reflections from each participant. The prompt for the first written reflection was designed to provide a broad opportunity for the participants to reflect on teaching and learning math. We include the extended prompt for this reflection below for context:

Before we ever decide to become teachers, we all have a long history in the field of education. Our experiences, whether good or bad, have helped to shape our perceptions of ourselves as learners first, then as potential educators. Each experience has also shaped our views about the subjects we’ve learned and our abilities to teach those subjects. This can be especially true of math. When it comes to this subject, there are many personal, academic, and cultural contexts that have influenced how we perceive our abilities. Some of these contexts we are very aware of, and others we may think about rarely or not at all. But all of them have influenced us as students and will affect how we teach. For this reflection, you will think about and examine your personal experiences in math. Consider not only your academic experiences but also your more informal exposure to these subjects at home and in your everyday life. Think and write about these experiences over time, from childhood to the present. What people and events encouraged you? Which experiences challenged you productively, and which made you just want to give up? Try to be as specific and as honest as you can as you explore how your experiences in this area have affected your current view of yourself as either “good” or “bad” or “other” at math.

Following this opening prompt at the beginning of the semester, the participants completed and reflected on five Problem Solving Exercises (PSEs) that were designed to foster deeper thinking about mathematical content for and with elementary-level students. In brief, the PSEs were assignments in which the participants were asked to engage with an elementary school student from their student teaching placements in a math activity (provided by the professors of the course). These math activities were conceptual and oriented toward building mathematical mindsets. The PSEs explored a range of mathematical topics and were specifically geared toward creating room for inquiry and reflection. For example, in one PSE the participants played Cross Out Singles with an elementary-grade student, which is a concept-driven game from Marilyn Burns’s book, About Teaching Mathematics (Burns, 2015). This game asks players to roll a die and strategically place numbers on a game board before adding the sums. A player wins the game by having the most duplicate addends, thereby reinforcing the strategy of placing numbers in an addition expression such that the sum is equivalent to another sum on the player’s board. In another PSE, the participants played a game that we titled Make Your Own Design with a student. This activity invites players to use 3–4 colors in creating an artistic image drawn on a 5x5 grid (for lower grades) or 10x10 grid (for upper grades). After the student has created the image, the participant works with the student to count the number of squares for each color and then represent the number as a fraction, decimal, and percent.

Throughout the semester, after the participants completed each PSE, they were asked to write a two-to-three-page reflection that addressed four elements of their experience. First, they were asked to make connections between the PSE and two of the main texts for the course (Mathematical Mindsets by Jo Boaler and About Teaching Mathematics by Marilyn Burns). Second, they were asked...
to identify and discuss the concepts that the PSE would introduce or reinforce for elementary-level students. Third, they were asked to identify and explore convergences and/or divergences between aspects of the PSE, tenets of the course texts, and observations from their field experiences. And fourth, participants were asked to make suggestions and modifications to the PSE, thus asking them to evaluate the PSE for possible improvements.

At the conclusion of the semester, participants were again asked to reflect on their experiences in the course vis-à-vis their own mathematical understandings. The prompt that was given to participants was:

1) Has taking this class changed your thoughts when it comes to your abilities in mathematics and science? Do you see any shifts (positive or negative)? What people and events caused this change or no change?

2) Which experiences challenged you productively? Which experiences made you just want to give up?

3) What do you think about the PSEs you were asked to play with a student? What do you like or not like about them? Would you consider such an approach to teaching and learning in mathematics? Why or why not?

Participants completed all of the written reflections as part of their coursework and posted them to a web-based discussion board that was part of our online learning management system.

In total, from the first phase of our data collection, we gathered seven reflections from each of the 28 participants: a written reflection from the beginning of the semester, a series of five written reflections following each of the PSEs, and a written reflection from the close of the semester. Following this first phase of data collection, we identified six participants from the group of 28 with whom to conduct semi-structured individual interviews to understand their reflections more deeply. These six participants were identified using a criterion-based purposive sampling method (Marshall & Rossman, 2006), as we were interested in speaking with specific students who might generate more substantive data for our analysis. To that end, we identified two students who self-identified as having a high and positive shift in their levels of critical reflection and inquiry toward math, two students who indicated a small but positive shift, and one student who indicated there was no shift in her reflections and inquiry regarding mathematics. The chart below offers a detailed snapshot of our data sources across both phases of data collection:
Data Collection—Phase Two

In the second phase of our data collection procedures, we identified five participants who were suitable for the semi-structured individual interviews. The interviews took place at the conclusion of the course, after all written work had been completed and reviewed in our first phase of data collection. The individual interviews were audio-recorded with the participants’ permission and transcribed verbatim. By using semi-structured individual interviews as a principal method of data collection, we relied on two assumptions: first, that the instrument best suited to learn about human existence is another human, and second, that in-depth interviews allow researchers and participants to build meaning around experience (Kvale & Brinkmann, 2009). The interview guide for the individual interviews was informed by scholarship on teacher candidates’ attitudes toward mathematics instruction as well as questions that arose from our initial coding of all of the data collected in Phase One. The questions elicited the participants’ reflections and opinions on three areas: a) mathematical insights that they may have gained from the course; b) reflections on their understandings of mathematics teaching and learning during the course; and c) areas of mathematical thinking that were still “stuck” in older (or original) ways of thinking.

Data Analysis

Our data analysis proceeded in multiple phases. At the conclusion of first phase of data collection, we began a first review of the total set of written reflections. Guided by the work of Saldaña (2016), we used verbatim coding, which is particularly useful in educational research as it honors the voices of the participants with their actual words. To do this, we highlighted portions of written reflections in which words, phrases, and short examples struck us as germane to the research question. From this initial analysis, we identified a few common phrases and examples that became possible themes to examine more closely. Additionally, we used this initial analysis to develop our interview protocol for the semi-structured interviews that we conducted in Phase Two. After all data (written reflections and interviews) were collected, we engaged in a second level of coding to enhance the findings. To this end, we used pattern (or descriptive) coding to start generating themes and working on interpretations of the data. Rather than create categories before the data analysis began, we used an inductive approach to discover patterns as we identified common refrains and repetitions in the data (Lawrence-Lightfoot & Hoffmann Davis, 1997; Saldaña, 2016).

In order to keep our emergent themes connected to the research questions and begin the process of researcher reflexivity, we wrote analytic memos that reflected our coding processes, our coding choices, and our emergent themes (Saldaña, 2016). These analytic memos pointed to segments of
written reflections and interview transcripts that coalesced around an idea and allowed us to interpret potential patterns and themes. We shared our analytic memos as researcher notes with one another. Initially, we identified a small number of potential patterns that we wanted to explore in more depth. In subsequent reads of the reflections and analytic memos, we looked for confirming and disaffirming data to rework the patterns or disrupt an emerging pattern. This process resulted in the development of three themes that were resonant with our research questions as well as the extant literature on teacher candidates’ reflections about teaching and learning mathematics. In the sections that follow our methodology, we present these themes supported by data, and we discuss the implications and applications of our findings.

**Limitations**

We believe the data presented are rich and worthy of continued consideration, but we also recognize that all studies are accompanied by limitations. First, it is important to state that the small number of participants does not lend itself to generalizability. This study was an in-depth inquiry into the critical reflections and insights of teacher candidates toward mathematics teaching and learning. While we believe the findings are relevant for other teacher educators, the small sample size and particular context limit a broad-spectrum application of findings from this study. Second, the present findings indicate that teacher candidates have a variety of views regarding teaching and learning mathematics. While this study offers insight into the reflections of a group of elementary teacher candidates, an in-depth examination of how these reflections are formed and how they evolve during content courses could provide insight into the factors that lead to critical inquiry and reflection.

**Findings**

In this section, we present findings from the data collected in this study. Overall, our data analysis revealed that our participants developed capacities for critical reflection and inquiry with regard to teaching and learning mathematics; however, there are still significant hurdles to overcome in preparing teacher candidates to teach math in a way that foregrounds inquiry and reflection. In the subsections that follow, we present data that support three findings from our data analysis: (a) reflecting on the past to understand the present; (b) teacher candidates’ experiences working with the problem solving exercises (PSEs); and (c) teacher candidates’ lack of clarity on next steps regarding their own growth in teaching mathematics.

**Reflecting on the Past to Understand the Present**

Our initial reflections asked participants to recall their experiences with mathematics from earlier in their school careers. Overall, our participants expressed positive feelings about math in the early grades but expressed that shifts toward negative feelings tended to happen starting in middle school and lasting through college. One participant shared:
If I had to pick a specific time where I felt the most confident with math, I would have to say elementary school. It was a time where there were no restrictions on time, questions, or imagination. If a student wasn’t understanding the problem, the teacher would go out of their way to reword the problem so the student would understand it better. All of this stopped in high school. All of a sudden, we had to learn a full lesson of math in under 40 minutes which didn’t leave any time for questions or one on one time with the teacher. Everything seemed rushed and if you didn’t finish in time, you simply had to teach yourself. Only it wasn’t so simple.

This participant’s reflection indicated that the learning context provided in elementary classrooms helped them enjoy mathematics because mathematics was made more accessible and meaningful.

In contrast to positive experiences recalled from elementary school, many of the participants expressed developing a negative perception about math while in high school, describing it as “lacking passion,” “a roller coaster ride,” and a subject that came with a “feeling of hatred.” The participants’ reflections suggested that certain teacher behaviors increased math anxiety, including presenting the content with a negative attitude, failing to clarify the content, and showing a lack of understanding for those who needed extra time to grasp difficult math concepts. By contrast, the participants who expressed positive experiences with mathematics all through school often identified themselves as “math people.” In other words, when a participant noted positive experiences with mathematics across grade levels, it was often associated with their “ability” to understand mathematics rather than the experiences provided to them.

Our analysis of the participants’ initial reflections also revealed that their struggles centered on a lack of understanding of the concepts because they felt their teachers or parents did not help them make sense of mathematics. One of the participants reflected on her experiences with math in a high school classroom:

The teacher started writing a problem on the board and was “teaching” us how to do it while we were unpacking our bags. We didn’t even have our notebooks out. After doing that problem on the board, she would hand out a worksheet that had similar problems and that was our lesson for the day. Then she would go sit at her desk and read a magazine and not instruct us any further. When we would ask for help she would tell us to look at the problem on the board but nobody understood her process. The only reason I passed that class was because she gave points for doing basic things like handing out pencils. After this class my hatred for math did not let me succeed in the other areas like trig and pre-calc and it didn’t help that those teachers also did not teach. Math in and after high school was horrible.

By contrast, participants associated positive feelings with mathematics when the teachers and parents made the math content accessible and fun. One participant said, “I enjoyed writing proofs and learning all of the rules for proving triangles congruent. I attributed this continued enjoyment of math to the teacher I had at the time. She was stern, but always made the classroom a fun and lively environment to learn the subject.” Another participant mentioned that her father helped her uncover mathematics:
My dad was the one who really encouraged us to keep working at the problem. He showed us time after time that if we kept thinking, kept working at the problem, and used our resources, we would be able to solve any problem. My dad constantly encouraged us to figure out our own ways to solve the problem, even if it was not what the teacher had taught us. Math was creative and exciting and possible.

Data from our final reflection of the semester pointed to the fact that the teacher candidates’ understandings about who can “do” mathematics evolved. And more specifically, there was a new understanding about who could teach and learn mathematics. Provided with an opportunity to reflect on their journeys in the course, participants offered emotional responses indicating that early experiences can have a lasting impact on one’s teaching and learning mathematics. One participant shared:

When I was younger, I learned in an environment in which the first person to get the correct answer understood the material best and if someone got it wrong, even for a moment, they were a slow learner, therefore not as smart. In addition, the environments I learned in as a small student were mostly teacher centered. This meant that the teacher knew best and could always give the right answer when a student was not achieving the right answer. To un-learn this idea and educate myself on the benefits of mistakes and growth from error was similar to forgetting an instinct.

In another reflection, a participant indicated a change in perception by stating:

Taking this class has definitely changed my thoughts when it comes to my math abilities. I always had the fixed mindset that I just couldn’t do math and didn’t have any number sense. Now I see that you have to reason through each math problem, and that [there] are different ways to solve one equation. I see my shift as a positive one and that now I have a growth mindset for my math abilities and children’s capabilities.

The majority of the participants indicated that reflecting on their prior experiences with mathematics as they learned about the math content allowed them to redefine what counts as mathematics teaching and learning. Comments such as the following were common among the data: “Prior to this course, mathematics was a subject that I dreaded. Over the course of this class, I was able to realize where this dread came from.” The following reflection is another example: “I came into the class thinking that math was about accomplishing the correct answer through a singular path. However, through the continuance of the class, I was able to see that math is more about thinking and understanding than answers and scores.” As Skott (2011) states, teachers’ prior experiences often lay the foundation for their own practice as teachers of mathematics. Throughout this course, we aimed to offer our participants the opportunity to develop a reflective approach to teaching and learning mathematics. Overall, the data presented in this section point toward a small development in the teacher candidates’ perceptions about what math “is” and who can “do” math. Specifically, from the beginning of the study to the end, the data revealed that the participants went from thinking about math as “getting a correct answer” to seeing mathematics as problem solving, reasoning, curiosity, and enjoyment. We find these results encouraging because several studies have repeatedly found that teacher confidence in mathematics is highly correlated with students’ learning and students’ confidence in themselves as mathematics learners (Stipek et al., 2001).
Experiences Working with the Problem Solving Exercises (PSEs)

In general, the data revealed that the PSEs and accompanying opportunities for reflective thinking allowed the participants to grow their understandings of math in a way that was different and new for them. One participant commented, “I really liked the PSEs because they were a different way to do math. I’m going to bring more of those problem solving exercises into the classroom that I’m teaching.” It was common for us to encounter participants saying something similar to this throughout our data analysis. Frequently, participants shared that the PSEs were a “different way to do math” for them and that the work they did in the PSEs was new and engaging. To add to this, here is another excerpt from a participant:

\[
\text{At the beginning of the semester, I was thinking of math as something that has to be done. Like, the most creative way you could do it was “If I have cookies and you have cookies and now we’re sharing our cookies,” you know? I didn’t see math as fun in any way. I saw it really as this chore. The class, especially the texts and the exercises ... I was able to obtain knowledge and obtain ideas and ways to be creative and ways to be free.}
\]

This idea—that the PSEs presented a math in a creative or “free” way—was common throughout the reflections and interview responses. Overall, we found that participants were surprised they could engage in mathematical thinking in this new way and also pleased to have something refreshing and fun to think about with regard to math. These data points illustrate the participants’ ability to express new mathematical knowledge (Ball, 1990) as a result of engaging with the inquiry-based PSEs and the associated reflective practice. Specifically, the participants expressed that the process of doing the PSEs led to a view of mathematics that was less static (e.g., “a chore”) and more dynamic (e.g., “creative” and “free”) (Brophy, 1991; Stipek et al., 2001).

Another finding that emerged in our analysis of the reflections on the PSEs was less generative: we found that participants were somewhat rigid in their approach to the PSEs, in particular when they were not sure how to adapt a PSE to a student in a different grade level. One PSE assignment (Make Your Own Design) asked the student to use multiple colors to draw an desired image on a 10x10 grid and then evaluate that image for fractional portions of color. One of our participants attempted this PSE with a younger student and shared the following reflection:

\[
\text{I remember looking at the PSE and thinking “Uh, ok.” I had my student color it in but the thing is, they had no idea what they were doing. They just thought they were coloring. And I was like, “Oh, color in the boxes whatever you want.” It didn’t register that it was math for them. It was just like, coloring.}
\]

This comment demonstrated a rigidity that was common to some, but not all, of the participants in our course. With all of the PSEs, there was a need to contextualize the work, scaffold the learning, and support the young learner in exploring the math concept. We found that some of our own students were not able to see the broader learning outcomes for some of the PSEs and fell into traps of rigid thinking wherein they felt that the PSEs were too challenging for their students. Additionally, this was reinforced by a common refrain we saw in the reflection data: many of our participants felt that the PSEs relied too much on luck. Although luck is involved in rolling dice and
generating numbers (as for certain PSEs that required spinners or dice), strategy is needed to follow that luck if a player is going to win the game by placing digits in certain advantageous spots. We were baffled by the recurrent use of the word “luck” that our participants shared and found that in general, the participants displayed a lack of awareness of how to contextualize advanced strategic thinking for themselves and their students.

**Lack of Clarity on Next Steps**

While we found that the PSEs had a broadening effect on the participants’ conceptions of what mathematics teaching could look like, the participants still expressed a lack of deeper understanding of mathematical content knowledge. Additionally, they were not aware of what they needed to do to support themselves on their journeys to becoming stellar math educators. Data from the individual interviews revealed that participants broadened their conceptions of what math could hold for themselves as well as their students as a result of engaging with the PSEs. Participants expressed enthusiasm around their students’ excitement to play math games with them. One participant shared, “I enjoyed playing the games with my students, they were in fun mode. They were like, ‘Yes, we can play!’ and I think to them, it’s like flipping education.” Another participant described her experience playing the game within the PSE with her student: “It was really nice because I asked a student [to play] and they were like, ‘Yeah, sure!’ and they were bubbly about it, which is really nice. I really liked that just because it was a moment in math that I hadn’t really experienced before because I don’t really have any experience of math through games.” These statements were representative of the finding that by engaging in the inquiry-based PSEs with their students, the participants broadened their conceptions of math activities, saw the subject “more dynamically,” and “represented it in more varied ways” (Brophy, 1991, p. 352). Qualitative data obtained in this study complemented the results of other studies (Jao, 2017) by suggesting that experiencing new approaches to mathematics teaching and learning and having the opportunity to apply them in the classroom are important to shifting teacher candidates’ formed beliefs around mathematics.

We found that this dynamic view of mathematics teaching and learning happened on two levels. First, the participants expressed knowledge and feelings about mathematics that were positive (Ball, 1990; Quinn, 1997). One participant shared, “Students are succeeding not when their hands are shooting up and answering questions right away but when they are connecting it to things that they know already or other types of concepts that maybe [they’re] building on.” Another participant noted, “I don’t think it’s like ‘you get or you don’t.’ I used to think that but I really think that if you don’t get it today, you’ll definitely get it tomorrow, you know?” And a third participant stated, “I think the roadblocks come in the form of standardized tests because they restrict the way that teachers can teach math. Because it reiterates another roadblock, which is the focus on answers and correct answers.” In particular with phrases such as “I used to think that,” the participants were pointing to evidence of reconsidering firmly held beliefs and practices about how mathematics was taught and learned (Moon, 2004).
Despite such changes, participants still exhibited a lack of deeper understanding of mathematical content knowledge. While participants were enthusiastic about the possibilities that were afforded by using the PSEs with students, they were also entrenched in old—or early—ways of thinking about math. Across the board, the participants described their own elementary mathematical encounters as ones that were filled with rote work and a great deal of apprehension. These tethers to abstract memorization and drilled information came through most strongly for the three interview participants who indicated experiencing moderate or no shifts in their experiences with math. One participant critiqued her first-grade students for having failed to memorize addition facts: “They did not have those simple facts down at all. A lot of them were counting on their fingers, and some of them needed manipulatives which was shocking for me, because they couldn’t simply do the small numbers like that.” Another participant equated success with speed as her class got to the point where they could move faster: “Having them draw it out and having them explore that in different ways has been successful because we’re not wasting time now when the kids are stumped.” These participants’ comments illustrate firmly entrenched ideas about manipulatives (including fingers) as a crutch instead of a developmentally appropriate support, or “stumped students” as an impediment to finishing a lesson rather than an opportunity to explore misconceptions and create growth. In our initial passes through the interview data, we were surprised—and saddened—by comments such as these that we flagged as examples of fixed mindset (Boaler, 2015) thinking on the part of the teacher candidates. While the PSEs may have provided space for the participants to develop critical reflection and inquiry regarding mathematics instruction, there were firmly entrenched ideas about abstract memorization, speed, and accuracy that could not be shaken in a semester-long course.

Finally, we were surprised to find that on the whole, participants were not aware of what they needed to do to improve their math content knowledge and support themselves on their journeys to becoming impactful math educators. We ended each individual interview with the question, “You talked about some new learnings that you had as a result of working with the PSEs (and math in general) in new ways this semester. What else do you think you need as a future educator to teach math in these generative ways that you spoke about?” Naively, we expected—or hoped—that participants would share books they wanted to read, ideas they wanted to explore, and questions that were still unanswered. But largely, the data revealed that participants were caught off-guard by this question and that they did not have concrete or novel ways to answer it. One participant responded, “I need ... what do I need to support me? Um, I guess a breakdown of how to approach a different way, when a lesson is not, like—what do I— you know what I mean?” Another participant replied, “I think I would have to do a lot more practicing,” but it was unclear exactly what she meant by practicing, even upon further follow-up within the interview. A third participant said, “I guess I still need workshops and to continue doing professional development days and talking to other math teachers to see how they conduct math in a similar way or in an even more progressive way.” Finally, one participant responded, “I definitely think as far as support, I’ll always need support or new ideas. I don’t want to keep using the same ideas I use now, like 20 years from now. But I think as far as improving the ways I teach math, or the ideas I have on teaching math, I think that the texts definitely did help. Like I said, I would photocopy stuff from the text and use it in the classroom.” Evaluating all of the interview responses together, we found that on the whole, the participants provided a much narrower range of articulate and understandable responses to the question of what they would do next to develop themselves as math educators. Whereas responses to other
interview questions elicited detailed comments and illustrative examples, our participants were tongue-tied or vague when describing their next steps in professional growth.

**Discussion and Future Inquiry**

Teacher education is crucial to the quality of teaching in many subjects, especially when it comes to providing quality teaching and learning experiences in mathematics (Endacott & Sturtz, 2015; Roberts-Hull et al., 2015). This study sought to explore teacher candidates’ experiences in a mathematics methods course that explicitly foregrounded inquiry and reflection. Specifically, we were interested in understanding whether the participants were able to construct new meaning, conceive new perspectives, or challenge previously held assumptions about teaching and learning mathematics. In this section, we offer a discussion of the findings and suggest areas for future inquiry.

Teachers’ feelings toward mathematics can vary drastically, yet they are often influenced by how they were taught as students (McNeal & Simon, 2000; Hattie, 2003). In this study, the participants’ feelings toward mathematics seemed to originate from their personal experiences, experiences with schooling and instruction, and experiences with formal mathematical knowledge. These experiences tended to influence their thinking and behavior, especially in relation to their teaching practices and instructional approaches to mathematics. At the beginning of the semester, the participants expressed an understanding of mathematics as something that needs to be memorized rather than something that can be explored and investigated. These findings echoed Vinter (1999), who found that many preservice teachers have a limited understanding of mathematics rather than a real-world understanding of mathematics applications. He further claimed that this abstract understanding impacts teachers’ ability to make math content accessible and meaningful for their own students. Thus, mathematics instruction becomes more rigid when teachers feel bound to the textbook rather than free to provide more flexible and developmentally appropriate teaching and learning experiences (Ma, 2010).

In addition, the results of this study revealed that negative experiences with mathematics appear to be more prevalent than positive ones. It seems that for most of the participants, positive thinking about mathematics requires some effort, whereas negative thinking comes easily and often uninvitedly. In particular, participants’ previous negative experiences with mathematics hamper their relationship with mathematics as learners and teachers (Ball & Bass, 2000). The majority of the participants who indicated that they had negative experiences with mathematics growing up reported avoiding mathematics as they continued with their education. This avoidance exhibited itself with postponing advanced mathematics courses or not taking them at all. Far from starting from scratch, all kinds of echoes from the participants’ own negative experiences with mathematics as learners influenced their approach to teaching mathematics. These “negative” influences from the past, almost like ghosts, can appear in off-the-cuff remarks as simple as “I have never been a math person!” or in subtler, more deeply ingrained patterns of behavior, such as avoiding teaching mathematics. According to Shulman (1986), a change in teachers’ approach to teaching and
learning requires “an act of reason, continuing with a process of reasoning, culminating in performance of imparting, eliciting, involving or enticing and is then thought about some more until the process can begin again” (p.13). Our results revealed that after spending a semester reflecting on and exploring mathematics in meaningful ways, teacher candidates began to question the underlying reasons behind their reluctance to learn mathematics as students and teach mathematics as teachers.

A second discussion point related to these findings is that despite the shifts we observed with our participants, they seemed to fail to fully understand how to teach mathematics in a conceptually driven and exploratory way. In short, they were able to talk the talk but not walk the walk. Our data revealed that participants spoke of math in terms of dread and anxiety when they began the course and when they were asked to reflect on their earlier experiences as elementary school and high school students. Following a focus on exploration, inquiry, and reflection in this course, we noticed a shift in discourse coming from the participants. In the final reflections and in our post-course interviews, we heard language that indicated a greater acceptance of mathematics as a subject that invites exploration and inquiry and less rigid thinking about mathematics as a subject that required finding a right answer.

As professors and researchers in this study, we were curious about the impact of the PSEs within this course. We found that many of the participants did not see the math in the PSEs, instead explaining away some of the thinking involved as a component of luck. Further, we found that participants were used to more structured assignments, both as students of math themselves and as teacher candidates within our larger teacher preparation program. Thus, we found that the participants were able to engage with the PSEs on the level of a task, and they were able to reflect on these activities with appropriate language around conceptually driven mathematics; however, they were not able to see or discuss the underlying math constructs that the PSEs were trying to explore. While disappointing, this stands to reason as a finding given that the intervention within this study only spanned the course of one semester.

To conclude, we offer three items for future inquiry and next steps. First, in examining our research design, we acknowledge that our initial reflection asked participants to reflect on their prior experiences, which exposed a plethora of negative experiences. A future iteration of this study should ask participants to reflect on what they would have needed both in previous experiences and from a course-based intervention in order to foster new, more positive experiences. This level of insight might help craft course content that would have a more significant impact on attitudes and understanding among teacher candidates. Second, we believe this work could be augmented by collecting more quantitative data, potentially using tools that measure math anxiety among teacher candidates. This type of data would add to our qualitative findings and, considering a mixed-methods approach, would add additional layers of examination into this study. Third, the nature of our intervention was quite limited in time. We believe our findings point to worthwhile inquiries and reflections that teacher candidates can explore in order to develop a critical and inquiry-driven stance in their own understanding of teaching and learning mathematics. However, for most teacher candidates, a one-semester course is not enough to actualize this change in thinking. More
time with these kinds of concepts is needed, and a study that examines this would be a welcome addition to the field.

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